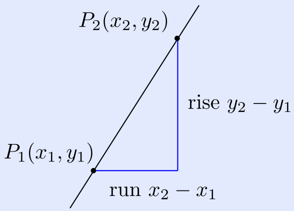
1. Line

1.1. Slope

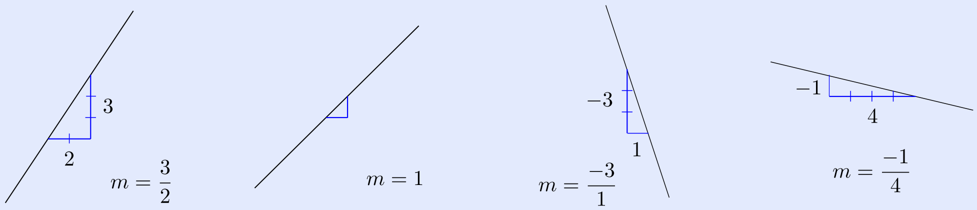
The **slope** m of a nonvertical line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is defined to be the ratio of the rise to the run:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$



This definition does not depend on the choice of the points P_1 and P_2. However, consistency is required, by which is meant, the first terms in the numerator and the denominator must come from the same point (otherwise the sign will end up being wrong).

It is useful to be able to estimate slopes of lines. This can be done by imagining a right triangle held against the line with tick marks on the legs. Any convenient spacing of the ticks can be used as long as it is consistent.



There are two special cases:

- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.

1.2. Point-slope form

We can write an equation for the line L through the point $P(x_0, y_0)$ having slope m by using the **point-slope form** of the line:

$$y - y_0 = m(x - x_0).$$

1.2.1 Example Find an equation of the line with slope -2 that passes through the point $(-1, 7)$.

Solution Using the point-slope form, we get $y - 7 = -2(x - (-1))$. □

Here is the reason the point-slope form works: A point (x, y) lies on L if and only if, using

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that point and the given point $P(x_0, y_0)$ to compute slope, we get m:

$$\frac{y - y_0}{x - x_0} = m.$$

Clearing the denominator gives the point-slope form.

If, instead of a point and a slope, we are given two points and are asked to find an equation of the determined line, we can first use the two points to find the slope as in 1.1 and then use the point-slope form with either of the two given points playing the role of $P(x_0, y_0)$.

1.3. Slope-intercept form

The **slope-intercept form** of a line L is

$$y = mx + b.$$

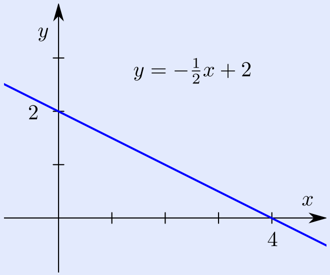
Here, m is the slope of the line L and b is the line's y-**intercept**, which is the y-coordinate of the point where the line crosses the y-axis (verified by observing that the point $(0, b)$ satisfies the equation).

1.3.1 Example Write the slope-intercept form of the line L that passes through the points $(6, -1)$ and $(-2, 3)$ and sketch.

Solution The slope of L is

$$m = \frac{3 - (-1)}{-2 - 6} = -\frac{1}{2},$$

so using the point-slope form we get $y - (-1) = (-1/2)(x - 6)$. Solving for y, we get the slope-intercept form $y = (-1/2)x + 2$, which has graph



(passes through y-axis at 2 and rises -1 for every run of 2). □

1 – Exercises

1 – 1 Find an equation of the line that passes through the points $(-1, 5)$ and $(3, 4)$.

1 – 2 Find an equation of the line with x-intercept 3 that is parallel to the line $2x - 4y = 1$.