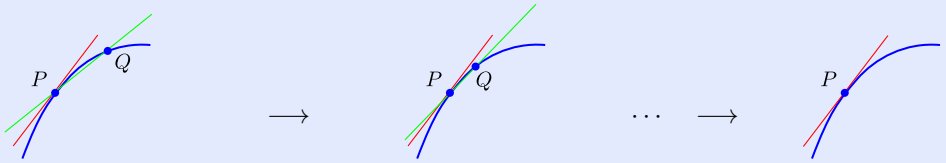
0. Introduction

Here are three examples to illustrate the idea behind calculus.

Decimal equivalent of fraction. The fraction $1/3$ can be approximated by using the decimal 0.3. An even better approximation is obtained by using the decimal 0.33. In fact, the more 3's that are appended, the better the approximation becomes. Calculus provides a way to push this process to the limit. The result is written $0.333\ldots$ and it is exactly $1/3$.

$$0.3 \longrightarrow 0.33 \cdots \longrightarrow 1/3$$

Slope of tangent line. In order to find the slope of a line, one needs two points on the line (the slope is the difference of their y-coordinates over the difference of their x-coordinates). This makes it difficult to imagine how to find the slope of a line tangent to a curve as in the figure on the right; the point P is one point on the tangent line, but there is no obvious way to get a second point.



In the figure on the left, the line through the points P and Q, called a secant line, is not too far away from being the tangent line, so its slope is taken as a first approximation to the desired slope. Moving Q closer to P provides a better approximation (middle figure). As Q gets ever closer to P, the slope of the secant line gets ever closer to the desired slope of the tangent. Calculus provides a way to push this process to the limit so that the exact slope of the tangent line is obtained.

Area of region. Elementary geometry provides no formula for the area of an odd shaped region such as that shown on the right.

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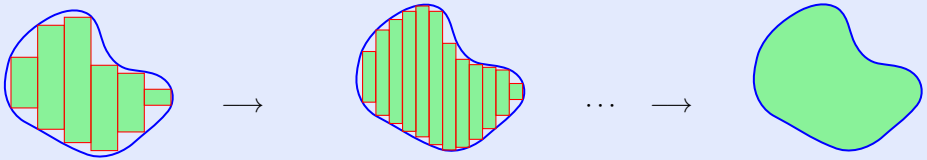

It does, however, give us a formula for the area of a rectangle (length times width). In the figure on the left, the sum of the rectangle areas is nearly the area of the region, so it is taken as a first approximation. Using more rectangles provides a better approximation (middle figure). As the number of rectangles gets ever larger, the sum of the rectangle areas gets ever closer to the area of the region. Pushing this process to the limit–again using calculus–we obtain the exact area of the region.

The three main objects of study in calculus are

- infinite series,
- derivative,
- definite integral,

and the examples given above illustrate each (in order).

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