

18 Derivative of exponential function

In this section, we get a rule for finding the derivative of an exponential function $f(x) = a^x$ (a , a positive real number). In particular, we get a rule for finding the derivative of *the* exponential function $f(x) = e^x$.

18.1 Statement

DERIVATIVE OF EXPONENTIAL FUNCTION. For any positive real number a ,

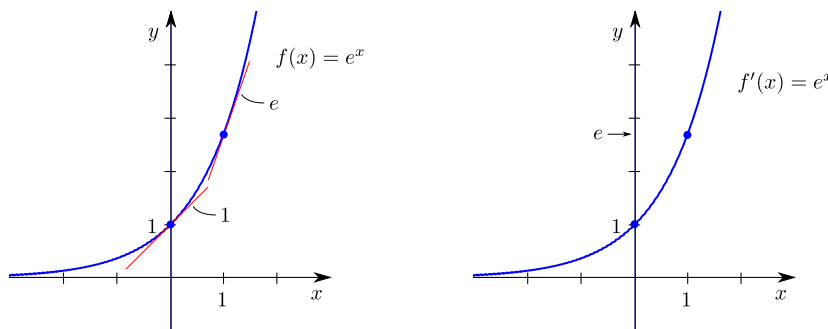
$$\frac{d}{dx} [a^x] = a^x \ln a.$$

In particular,

$$\frac{d}{dx} [e^x] = e^x.$$

For example, $\frac{d}{dx} [2^x] = 2^x \ln 2$. The second formula follows from the first, since $\ln e = 1$. In modeling problems involving exponential growth, the base a of the exponential function can often be chosen to be anything, so, due to the simpler derivative formula it affords, e is the base of choice.

The rule stated above is verified by using the definition of the derivative. Although we omit the details, we will at least indicate the reasonableness of the formula for the derivative of e^x by considering graphs. The formula says that the function $f(x) = e^x$ has general slope function $f'(x) = e^x$:



The height of the graph of the derivative f' at x should be the slope of the graph of f at x (see 15). This appears to be the case for the choices $x = 0$ and $x = 1$ as indicated. As x goes to $-\infty$, the slope of the graph of f approaches 0, so, as x goes to $-\infty$, we expect the height of the graph of f' to approach 0 as

well, and this is the case. Similarly, as x goes to ∞ , the slope of the graph of f approaches ∞ , so, as x goes to ∞ , we expect the height of the graph of f' to approach ∞ as well, and this is the case.

18.2 Derivative of exponential versus power rule

Although the functions 2^x and x^2 are similar in that they both involve powers, the rules for finding their derivatives are different due to the fact that for 2^x , the variable x appears as the exponent, while for x^2 , the variable x appears as the base:

$$\frac{d}{dx} \overset{\text{variable}}{\downarrow} [2^x] = 2^x \ln 2, \quad \frac{d}{dx} \overset{\uparrow}{\text{variable}} [x^2] = 2x^1.$$

(The same distinction holds if 2 is replaced by any real number.)

18.2.1 Example Evaluate $\frac{d}{dx} [5^x + x^5 - 3e^x]$.

Solution We use the sum rule and the constant multiple rule to break the derivative down into derivatives that we have rules for:

$$\begin{aligned} \frac{d}{dx} [5^x + x^5 - 3e^x] &= \frac{d}{dx} [5^x] + \frac{d}{dx} [x^5] - 3 \frac{d}{dx} [e^x] \\ &= 5^x \ln 5 + 5x^4 - 3e^x. \end{aligned}$$

□

18–Exercises

18–1 Find the derivatives of the following functions:

(a) $f(x) = 4e^x - 3^x + 5x^2$

(b) $f(t) = 7^t - 6\sqrt[3]{t} + 8e^t$